1

T/T^{MF}_{BCS}

0

1

I

II
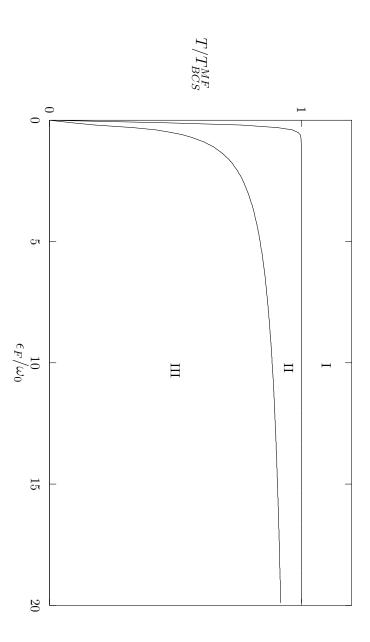
III

$\frac{\epsilon_F}{\omega_0}$

0

5

10

15

20

1

On the theory of the pseudogap formation in 2D attracting fermion systems

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Abstract

Two-dimensional system of the fermions with the indirect Einstein phonon-exchange attraction and added local four-fermion interaction is considered. It is shown that in such a system at resulting attraction between particles a new nonsuperconducting phase arises along with the normal and superconducting phases. In this, called "abnormal normal", or pseudogap, phase the absolute value of the order parameter is finite but its phase is a random quantity. It is important that the new phase really exists at low carrier density only, i.e. it shrinks with doping increasing in the case of phonon attraction. The relevance of the results for high-temperature superconductors is speculated.

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1 Introduction

The problem of an adequate description of the physical properties of high-temperature superconductors (HTSCs) still remains one of the actual problems of the modern solid state physics. It is connected with some peculiar properties of HTSCs. Among them there are such as quasi-2D character of electronic (and magnetic) properties, a relatively low and changeable carrier density n_f and its influence on properties of HTSCs (see, for example, review [1]).

Nowadays, one of the widely discussed questions on HTSCs is the problem of the so called pseudogap (or spin gap if magnetic subsystem of HTSCs is taken into account) [2, 3, 4], which is usually experimentally observed as a loss in the spectral weight of quasiparticle (or spin) excitations in normal state samples with lowered carrier density n_f [5, 6, 7]. Corresponding underdoped samples reveal some specific spectral, magnetic and thermodynamic pecularities which still continue to be not sufficiantly understood now. Moreover, the striking difference between the low (underdoped) and high (overdoped) density regions in HTSCs is increasingly debated and is considered as one of the very central and key questions in physics of the cuprates [8, 9].

The possibility of experimental changing of n_f value in HTSCs puts a rather general theoretical problem of the description of the crossover from composite boson superfluidity (low n_f) to Cooper pairing (large n_f) when n_f increases (in other words, a description of the transition from the so called underdoped regime to the overdoped one). Such a crossover was already studied for 3D and quasi-2D systems (see reviews [10, 11]). 2D case has been considered for the present at temperature $T = 0$ only [10, 12] what is connected with the Hohenberg-Mermin-Wagner theorem which forbids any homogeneous (long-range) order in pure 2D systems at $T \neq 0$ due to the long-wave fluctuations of the charged order parameter (OP).

The problem of the inhomogeneous condensate (Berezinskii-Kosterlitz-Thouless, or BKT, phase) formation was also considered despite of some difficulties in 2+1 relativistic field models [13] where the fermion concentration effects are irrelevant. At the same time these

effects were studied in nonrelativistic model in [14], for example, without taking into account the existence of the neutral OP ρ. Its consideration proves to be very important (see [15]) and results in the formation of a separate equilibrium phase with $\rho \neq 0$ which is located on the phase diagram of a system between normal and superconducting (here - BKT) ones. Due to fluctuations of the OP phase this new state of a system is of course also non-superconducting.

In this paper an attempt is made to study the crossover as well as the above mentioned new phase formation possibility in 2D fermion system with both a more realistic indirect (phonon) and also a direct (local) four-fermion (4F-) interactions. Thus, the work is to a certain extent a specific and non-trivial generalization of the preliminary short communication [15] where this non-superconducting phase appearance was studied for 4F-case only and of the paper [16] where Fröhlich model was used for the investigation of the crossover at $T = 0$. As it will be seen in the boson exchange model (in contrast to the pure 4F-case), the new phase really exists when n_f is rather small what allows to relate this result to underdoped HTSC compounds. But actually it is interesting to take into account a more real situation with an indirect attraction and some kind of local repulsion which may correspond to the short-range (screened) Coulomb interaction between carriers. In general case we, however, suppose that 4F-interaction can be repulsive and attractive as well. Besides, the case of total repulsion allows to explore the fermion-antifermion (electron-hole) pairing channel which in spite physical difference can be formally described by the same manner.

2 Model and main equations

Let us choose the simplest Hamiltonian density in the form:

$$H(x) = -\psi_\sigma^\dagger(x) \left(\frac{\nabla^2}{2m} - \mu \right) \psi_\sigma(x) + H_{ph}(\varphi(x)) +$$

$$g_{ph}\psi_\sigma^\dagger(x)\psi_\sigma(x)\varphi(x) - g_{4F}\psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x)\psi_\downarrow(x)\psi_\uparrow(x), \quad (x = \mathbf{r}, t), \tag{1}$$

where $\psi_\sigma(x)$ is a fermionic field with an effective mass m and spin $\sigma = \uparrow, \downarrow$; μ is the chemical potential of the fermions which fixes n_f; $\varphi(x)$ is a phonon field operator, g_{ph} and g_{4F} are the electron-phonon and the 4F-interaction coupling constants, respectively. As we said

above, g_{4F} can be positive (fermion-fermion attraction) or negative (fermion-antifermion attraction); in (1) we set $\hbar = k_B = 1$.

In (1) H_{ph} is the Hamiltonian of free phonons which can be described by the propagator

$$D(i\Omega_n) = -\frac{\omega_0^2}{\Omega_n^2 + \omega_0^2},$$
(2)

where $\Omega_n = 2n\pi T$ (n is an integer) is the Matsubara frequency [17]. As it follows from (2), the propagator $D(i\Omega_n)$ has been chosen in the simplest form with ω_0 being the Einstein (dispersionless) phonon frequency. It was done because of several reasons: first, this propagator gives a possibility to integrate the equations obtained; second, it is precisely the optic phonon and quadrupolar exciton modes with their relatively weak dispersion are widely considered as exchange bosons that can contribute into the hole-hole attraction in HTSCs [1, 18, 19], and third, the qualitative results concerning retardation effects do not strongly depend on the model studied. But on the other hand, the propagator (2) for the model under consideration can hardly be used for quantitative description of the cuprates and also their spin-wave branches which, as it is well-known, have linear dispersion law.

It is important that the Hamiltonian (1) is invariant under symmetry transformations of two types [20], namely:

$$\psi_\sigma(x) \rightarrow \psi_\sigma(x)e^{i\alpha(x)}, \quad \psi_\sigma^\dagger(x) \rightarrow \psi_\sigma^\dagger(x)e^{-i\alpha(x)}$$
(3)

and

$$\psi_\uparrow(x) \rightarrow \psi_\uparrow(x)e^{i\alpha(x)}, \quad \psi_\downarrow(x) \rightarrow \psi_\downarrow(x)e^{-i\alpha(x)},$$

$$\psi_\uparrow^\dagger(x) \rightarrow \psi_\uparrow^\dagger(x)e^{-i\alpha(x)}, \psi_\downarrow^\dagger(x) \rightarrow \psi_\downarrow^\dagger(x)e^{i\alpha(x)}$$
(4)

which must be taken into account. The phase $\alpha(x)$ in (3) and (4) is real.

With the purpose to calculate the phase diagram of the system it is necessary to find its thermodynamic potential. It can be calculated by making use of the auxiliary bilocal field method (see, for example, [21]), which is a generalization of the standard Hubbard-Stratonovich one for the boson-exchange case. Then the grand partition function Z can be expressed through a path integral over the fermionic $\psi_\sigma(x)$ and the complex auxiliary fields (for example, $\phi(x, x') \sim \ <\psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x') >$).

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In the case of the model (1) it is convenient following the Ref.[22] to introduce the bispinor

$$\Psi^\dagger(x) = (\psi_\uparrow^\dagger(x), \psi_\downarrow^\dagger(x), \psi_\uparrow(x), \psi_\downarrow(x)) \tag{5}$$

and its hermitian conjugate one which here are the analogous of the Nambu spinors [23]. After the substitution of (5) in (1) the Hamiltonian takes the form:

$$H(x) = -\Psi^\dagger(x)(\frac{\Delta^2}{2m} + \mu)I \otimes \tau_z \Psi(x) - g_{ph}\Psi^\dagger(x)I \otimes \tau_z \Psi(x)\varphi(x) -$$
$$g_{4F}\Psi^\dagger(x)I \otimes \tau_z \Psi(x)\Psi^\dagger(x)I \otimes \tau_z \Psi(x) + \varphi(x)D^{-1}(x)\varphi(x), \tag{6}$$

where $I \otimes \tau_z$ is the direct product of the unit I and Pauli τ_z $2 \otimes 2$ matrices; $D(x)$ is defined by (2). In such a representation of the Hamiltonian (6) and the field variables (5) the Feinman diagram technique becomes applicable in its ordinary form [22]. Thus, after standard excluding of the boson field $\varphi(x)$, the Lagrangian of the system can be expressed by the formula:

$$L(x_1, y_1, x_2, y_2) = \Psi^\dagger(x)[-\partial_\tau + (\frac{\Delta^2}{2m} + \mu)I \otimes \tau_z]\Psi(x) -$$
$$\frac{1}{2}\Psi(x_1)\Psi^\dagger(y_1)I \otimes \tau_z K(x_1, y_1; x_2, y_2)\Psi(x_2)\Psi^\dagger(y_2)I \otimes \tau_z. \tag{7}$$

The kernel K is the effective non-local inter-particle interaction function and will be explicitly defined in the momentum space below.

In order to explore the pairing possibility in the system let us introduce the bilocal auxiliary field, or OP,

$$\phi(x_1, y_1) = \tau_z K(x_1, y_1; x_2, y_2)\Psi(x_2)\Psi^\dagger(y_2) \equiv iI \otimes \tau_y \phi_{ch}(x_1, y_1) + \tau_x \otimes \tau_x \phi_{ins}(x_1, y_1) \tag{8}$$

(the integration over x_2 and y_2 is assumed). Here $\phi_{ch} \sim < \psi_\uparrow^\dagger \psi_\downarrow^\dagger >$ and $\phi_{ins} \sim < \psi_\downarrow^\dagger \psi_\uparrow >$ are electron-electron (charged) and electron-hole (insulating) OP, respectively (we neglect non-zero spin pairing). The auxiliary fields ϕ_{ch} and ϕ_{ins} are responsible for the dynamical symmetry breaking (in according with (3) and (4), correspondingly).

Adding to (7) a zero term

$$\frac{1}{2}[\phi(x_1, y_1) - K(x_1, y_1; x_1', y_1')\Psi(x_1')\Psi^\dagger(y_1')]K^{-1}(x_1, y_1; x_2, y_2)[\phi(x_2, y_2) -$$

$$K(x_2, y_2; x_2', y_2') \Psi(x_2') \Psi^\dagger(y_2')]$$

to cancel the 4F-interaction, one could obtain the Lagrangian in the form:

$$L(x_1, y_1; x_2, y_2) = \Psi^\dagger(x_1)[-\partial_\tau + (\frac{\Delta^2}{2m} + \mu)I \otimes \tau_z - \phi(x_1, y_1)]\Psi(y_1) +$$
$$\frac{1}{2}\phi(x_1, y_1)K^{-1}(x_1, y_1; x_2, y_2)\phi(x_2, y_2), \qquad (9)$$

Let us transform the expression for the kernel K; then in the momentum space it is

$$K(x_1, y_1; x_2, y_2) =$$

$$\int \frac{d^3P d^3p_1 d^3p_2}{(2\pi)^9} K_P(q_1; q_2) exp\left[-iP(\frac{x_1 + y_1}{2} - \frac{x_2 + y_2}{2}) - ip_1(x_1 - y_1) - ip_2(x_2 - y_2)\right]$$

where $p_i = (\vec{p_i}, \omega_i)(i = 1, 2)$ and $P = (\vec{P}, \omega)$ designate the relative and the centre of mass momenta, respectively. By the definition the kernel $K_P(p_1; p_2)$ is in fact independent of P (so we omit index P henceforth) and aquires the simple form

$$K(p_1; p_2) = g_{ph}^2 D(p_1 - p_2) - g_{4F} \qquad (10)$$

which will be used in (9). The last expression evidently demonstrates that the total character of the effective inter-particle interaction as it always takes place in such a situation [24, 23] defined by the possible competition between the first (retarded) and second (retardless) terms in (10), or their common action.

The partition function can be written as:

$$Z = \int \mathcal{D}\Psi^\dagger \mathcal{D}\Psi \mathcal{D}\phi \mathcal{D}\phi^* \exp\left[-\beta \int L(\Psi^\dagger, \Psi, \phi^*, \phi)dxdy\right]$$

$$\equiv \int \mathcal{D}\phi \mathcal{D}\phi^* \exp(-\beta\Omega[\mathcal{G}]), \quad (\beta = 1/T),$$

where $\Omega[\mathcal{G}]$ is the thermodynamic potential which in the "leading order" is

$$\beta\Omega[\mathcal{G}] = -\text{Tr}\left[Ln\mathcal{G}^{-1} + \frac{1}{2}\text{Tr}(\phi K^{-1}\phi)\right], \qquad (11)$$

where Tr includes 2D spatial $\mathbf{r}$ and "time" $0 \le \tau \le \beta$ integrations as well as the standard trace operation. The full Green function of a system is

$$\mathcal{G}^{-1} = -\partial_\tau + \tau_z\left(\frac{\nabla^2}{2m} + \mu\right)I \otimes \tau_z + \phi. \qquad (12)$$

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From (11) and (12) we arrive to the ϕ-equation (the Schwinger-Dyson one):

$$\delta\Omega/\delta\phi = \phi - \int \frac{d^2\vec{k}d\omega}{(2\pi)^3} K(p; \vec{k}, \omega)\mathcal{G}(\vec{k}, \omega) = 0. \tag{13}$$

Substituting (13) into (11) one can obtain the expression for $\Omega(\mathcal{G})$:

$$\beta\Omega(\mathcal{G}) = -\mathrm{TrLn}\mathcal{G}^{-1} + \frac{1}{2}\mathrm{Tr}\mathcal{G}K\mathcal{G},$$

The last is the well-known Cornwell-Jackiw-Tomboulis formula for the effective action in the one-loop approximation [25]. Using (13) we can rewrite this expression in the form

$$\beta\Omega(\mathcal{G}) = -\mathrm{Tr}[\mathrm{Ln}\mathcal{G} + \frac{1}{2}[\mathcal{G}\mathcal{G}_0^{-1} - 1]]. \tag{14}$$

As it was shown by Thouless et al.[26] (see also [15]) in 2D case it is natural to pass to a new parametrization of the OP (8) - its absolute value and the phase, namely:

$$\phi_{ch}(x, y) = \rho_{ch}(x, y)exp[-i(\theta(x) + \theta(y))],$$

$$\phi_{ins}(x, y) = \rho_{ins}(x, y)exp[-i(\theta(x) + \theta(y))], \tag{15}$$

where ρ_{ch} and ρ_{ins} are real.

As it will be shown below, with the given kernel (10) there can arise only one (ϕ_{ch} or ϕ_{ins}) OP. Therefore, it is necessary to make, simultaneously with (15), the spinor transformation (in according with (3) and (4))

$$\Psi^\dagger(x) = \chi^\dagger(x)exp(i\theta(x)I \otimes \tau_z), \tag{16}$$

$$\Psi^\dagger(x) = \chi^\dagger(x)exp(i\theta(x)I \otimes \tau_z), \tag{17}$$

(the spinor $\chi(x)$ is real and formally corresponds to chargeless fermions). It is easy to see from (15), (16) and (17) that the phase dependences of the charged and insulating OPs are similar. Below we shall obtain θ-corrections for the ϕ_{ch} case, because the final equations for ϕ_{ins} will be the same up to substitution $\rho_{ch} \rightarrow \rho_{ins}$. The reason is that when $K(p_1, p_2)$ describes the attraction (charge pairing channel) the symmetry of the Lagrangian under operations (3) proves to be crucial for the representation (16); while when $K(p_1, p_2)$ corresponds to the repulsion (chargeless, or insulating, pairing channel) the symmetry (4) becomes already

important and the representation (17) must be used as a "working" one. With this difference the rest of the calculations are almost identical and so we shall consider in detail the charge channel which is most interesting for metallic (superconducting) systems.

In variables (16) the Green function (12) transformes to

$$
\begin{aligned}
\mathcal{G}^{-1} &= -\partial_\tau + I \otimes \tau_z \left(\frac{\nabla^2}{2m} + \mu \right) + iI\tau_y \rho_{ch} + \\
& \quad I \otimes \tau_z \left(\partial_\tau \theta + \frac{\nabla \theta^2}{2m} \right) + iI \otimes I \left(\frac{\nabla^2 \theta}{2m} + \frac{\nabla \theta \nabla}{m} \right) \equiv G^{-1}(\rho_{ch}) - \Sigma(\partial \theta).
\end{aligned} \quad (18)
$$

Then using (18) supposing θ gradients are small (the hydrodynamic approximation) and taking them into account up to the second order the effective potential (14) can be naturally divided it two parts: $\Omega = \Omega_{kin}(\rho_{ch}, \nabla\theta) + \Omega_{pot}(\rho_{ch})$ where in $(\nabla\theta)^2$ approximation

$$
\begin{aligned}
\beta\Omega_{kin}(\rho_{ch}, \nabla\theta) = \mathrm{Tr} \Big[& G\Sigma - G_0\Sigma + \frac{1}{2}G\Sigma G\Sigma - \\
& \frac{1}{2}G_0\Sigma G_0\Sigma + \tau_x \otimes I \frac{1}{2}i\rho_{ch}G(G\Sigma + G\Sigma G\Sigma) \Big].
\end{aligned} \quad (19)
$$

Assuming now that $\rho_{ch}(x, y)$ is homogeneous [1] after somewhat tedious but otherwise straightforward calculation one can obtain from (19):

$$
\Omega_{kin}(\rho_{ch}, \Delta\theta) = \frac{T}{2} \int_0^\beta d\tau \int d^2\mathbf{r} J(\mu, T, \rho_{ch}(\mu, T))(\nabla\theta)^2, \quad (20)
$$

where

$$
J(\mu, T, \rho_{ch}(\mu, T)) =
$$
$$
\frac{1}{2\pi}(\sqrt{\mu^2 + \rho_{ch}^2} + \mu + 2T \ln\left[1 + \exp\left(-\frac{\sqrt{\mu^2 + \rho_{ch}^2}}{T} \right) \right] -
$$
$$
\frac{T}{\pi}\left[1 - \frac{\rho_{ch}^2}{4T^2} \frac{\partial}{\partial(\rho_{ch}^2/4T^2)} \right] \int_{-\mu/2T}^\infty dx \frac{x + \mu/2T}{\cosh^2 \sqrt{x^2 + \rho_{ch}^2/4T^2}} \quad (21)
$$

plays the role of neutral OP stiffness. Note that in comparison with the retardation free 4F-model [15] the last expression contains one more term, namely: the term with the derivative.

The equation for the temperature T_{BKT} of the BKT transition can be written down after direct comparison of the kinetic term (20) in the effective action with the Hamiltonian of

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[1]Equations for ρ_{ch} and ρ_{ins} will be obtained below and, as was shown in [16], it is an admissible approximation to put in them the value ρ_{ch} (and ρ_{ins}) independent of spatial and time variables.

the 2D XY-model which has the formally identical form [27]. Hence it easy to conclude that

$$\frac{\pi}{2} J(\mu, T_{BKT}, \rho_{ch}(\mu, T_{BKT})) = T_{BKT}. \tag{22}$$

The essential difference of this equation obtained from that for the XY-model is its spontaneous dependence on μ (or n_f) and ρ_{ch}.

To complete the set of self-consitent equations which allow to trace an explicit dependence of T_{BKT} on n_f, the equations for ρ_{ch} and μ have also to be given. In particular, the equation for $\rho_{ch}(i\omega_n)$ is nothing else but (13) with $\nabla\theta = 0$, i.e. the Green function G of the neutral fermions substitutes $\mathcal{G}$, so that (13) in frequency-momentum represantation takes the form

$$\begin{pmatrix} \rho_{ch}(i\omega_n) \\ \rho_{ins}(i\omega_n) \end{pmatrix} = T \sum_{m=-\infty}^{\infty} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \begin{pmatrix} -\rho_{ch}(i\omega_m) \\ +\rho_{ins}(i\omega_m) \end{pmatrix} \frac{K(\omega_n, \omega_m)}{\omega_m^2 + \varepsilon^2(\mathbf{k}) + \rho_{ch}^2(i\omega_m) + \rho_{ins}^2(i\omega_m)}, \tag{23}$$

where $\omega_n = (2n+1)\pi T$ is the Matsubara fermionic frequency [23] and kernel $K(\omega_m, \omega_n)$ is defined above. We cited the final equations for both OPs, ρ_{ch} and ρ_{ins}, in order to show only that they indeed are the same but alternative if the kernel K changes the sign.

An analitical solution of these equations, as well as obtaining both the equation (22) and the number equation needed is only possible if one supposes that $\rho_{ch}(i\omega_n)$ does not depend on the Matsubara frequencies (see footnote on p.8.).

Making use of this approximation the number equation which follows from the ordinary condition $V^{-1}\partial\Omega[\mathcal{G}]/\partial\mu = -n_f$ (V is a volume of a system) and is crucial for crossover description has to be added to (22) and (23) for self-consistency; so one comes to

$$\sqrt{\mu^2 + \rho_{ch}^2} + \mu + 2T \ln\left[1 + \exp\left(-\frac{\sqrt{\mu^2 + \rho_{ch}^2}}{T}\right)\right] = 2\epsilon_F, \tag{24}$$

where $\epsilon_F = \pi n_f/m$ is the Fermi energy of free 2D fermions with the simplest quadratic dispersion law. Thus, in the case under consideration all unknown quantities ρ_{ch}, μ and T_{BKT} are the explicit functions of n_f.

3   Analysis of the solutions

Unlike the usual (with T-independed unit vector) XY-model, in the superconducting one there exist two critical temperatures: T_ρ where formally the complete OP given by (8) arises but its phase is a random quantity, i.e. $< \phi(x,y) >= 0$ [2], and another one, $T_{BKT} < T_\rho$, where the phase of the OP becomes ordered, so that $< \phi(x,y) > \neq 0$. In other words, the temperature T_ρ is in fact the temperature of appearance of neutral OP only which has discrete symmetry and thus is not at variance with general theorems. Recall, that according to the equations obtained above both these temperatures, what is important, directly depend on the carrier density in the system.

Critical temperature T_ρ can be found from (21)-(24) by putting $\rho_{ch} = 0$ (what in accordance with derivation of these equations corresponds, in fact, to the mean-field approximation). As a result, a 2D metal with temperature decreasing passes from normal phase $(T > T_\rho)$ to another one where averaged homogeneous (charged) OP $< \phi(x,y) >= 0$, or, what is the same, superconductivity is absent, but chargeless OP $\rho_{ch} \neq 0$. It is evident that the pseudogap is formed just in the temperature region $T_{BKT} < T < T_\rho$, because, as follows from the above formulas (see, for instance, (21)-(24)), $\rho_{ch} = \rho_{ch}(T)$ enters all spectral characteristics of 2D metal in the same way as the superconducting gap $\Delta(T)$ enters into corresponding expressions for ordinary superconductors. It justifies why this new phase can be called the "abnormal normal" phase or better pseudogap one. The density of states near ϵ_F in the pseudogap is definitely less than in the normal phase, but is not to be equal zero as in superconducting one. The latter has to be checked by direct calculation of the one-particle fermion Green function what is most likely a separate problem which we do not touched upon here.

The phase diagram of the system can be found from the equations (21)-(24). There are different behaviours of $T_\rho(n_f)$ and $T_{BKT}(n_f)$ for various correlation between interaction constants.

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[2]Because of ρ_{ch} and ρ_{ins} can not exist simultaneously (see (23)) the index ρ means the only OP, which appears at finite sign of total interaction.

1) $g_{4F} > 0, g_{ph} = 0$ (retardless interaction).

This case has been partly analyzed in Ref.[15]. It corresponds to fermion-fermion pairing due to the local interaction. Note (see (23)) that in the case of resulting attraction between fermions fermion-antifermion (or electron-hole) insulating pairing channel is absent, i.e. $\rho_{ins} = 0$. The phase diagram for this case is presented in fig.1. It shows that the abnormal normal phase exists at any concentration value n_f and the temperature width of this phase region weakly increases with n_f increasing, and BKT phase always begin to form when $\rho_{ch}(T_{BKT})$ is finite.

At $\epsilon_F \to 0$ the temperature of BKT phase formation is defined by equality $T_{BKT} = \epsilon_f/2$, and T_ρ as function of n_f can be found from the equation $T_\rho \ln(T_\rho/\epsilon_F) = W \exp(-2/g_{4F}m)$, which follows from (23) (W is the conduction band width).

2) $g_{4F} > 0, g_{ph} \neq 0$.

The situation here is almost the same as previous one. The presence of the indirect interaction leads to the effective growth of the effective 4F-interaction constant $g_{4F}^* > g_{4F}$ (see (23)). The latter in one's turn simply results in increasing of the region between T_ρ and T_{BKT} (fig.2) and keeps the form shown on fig.1.

3) $g_{4F} = 0, g_{ph} \neq 0$ (a pure indirect interaction).

This is one of the most interesting cases because it corresponds to the widely accepted electron-phonon (or BCS-Bogolyubov-Eliashberg) model of superconductivity. The numerical calculations of the phase diagram is presented in fig.2. This diagram shows that comparatively large region with the abnormal normal (pseudogap) phase exists at rather low carrier concentrations only and its temperature area shrinks at $n_f \to \infty$. Such a behaviour qualitatively agrees with that one which takes place in real HTSCs samples [5, 6, 7, 8] demonstrating that pseudogap (and spin gap also) region is observed in underdoped samples.

Indeed, it is not difficult to make certain that the assymptotics for $T_\rho(n_f)$ and $T_{BKT}(n_f)$ have the following forms:

i) when ratio $\epsilon_F/\omega_0 \ll 1$ (very low fermion density, or local pair case) the first of them satisfies the equation $T_\rho \ln(T_\rho/\epsilon_F) = \omega_0 \exp(-4\pi/g_{ph}^2 m)$ which immediatedly results in $\partial T_\rho(n_f)/\partial n_f|_{n_f \to 0} \to \infty$. At the same time the temperature T_{BKT} at $n_f \to 0$ has another

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carrier density dependence and as above $T_{BKT} = \epsilon_F/2$ what simply means that here it again is equal to the number of composite bosons; in this density region $T_\rho/T_{BKT} \gg 1$ (such an inequality is also correct for the pure 4F-interaction).

ii) in the case $\epsilon_F/\omega_0 \gg 1$ (very large fermion density, or Cooper pair case) one easily arrives to the standard BCS value: $T_\rho = (2\gamma\omega_0/\pi)\exp(-2\pi/g_{ph}^2 m) \equiv T_{BCS}^{MF} = (2\gamma/\pi)\Delta_{BCS}$ (Δ_{BCS} is the usual one-particle BCS gap at $T = 0$). In other words, the temperature T_ρ in this limit becomes equal to its BCS value [3]. The T_{BKT} asymptotics for this case is not so evident and requires more detailed consideration.

First of all, it is naturally to suppose that for large n_f value $T_{BKT} \to T_\rho$. Then it is necessary to check the dependence of ρ on T as $T \to T_\rho$. For that the equation (23) can be transformed to:

$$\frac{2\pi}{g_{ph}^2 m} = \int_0^\infty dx \left(\frac{\tanh\sqrt{x^2 + \rho_{ch}^2/4T^2}}{\sqrt{x^2 + \rho_{ch}^2/4T^2}} - \frac{\tanh\sqrt{x^2 + \rho_{ch}^2/4T^2} - \tanh(\omega_0/2T)}{2(\sqrt{x^2 + \rho_{ch}^2/4T^2} - \omega_0/2T)} - \right.$$
$$\left. \frac{\tanh\sqrt{x^2 + \rho_{ch}^2/4T^2} + \tanh(\omega_0/2T)}{2(\sqrt{x^2 + \rho_{ch}^2/4T^2} + \omega_0/2T)} \right) \tag{25}$$

(where it was used that in this concentration region, the ratio $\mu/2T_\rho \simeq \epsilon_F/2T_\rho \gg 1$ because of $\mu \simeq \epsilon_F$[10, 11, 12, 16]).

On account of usually $\omega_0/2T_\rho \gg 1$, only very small x give the main contribution to the integral (25) (it is seen from the limit $\rho/2T_\rho \to 0$ when $\epsilon_F/\omega_0 \to \infty$). Therefore the latter expression takes the approximate form:

$$\frac{2\pi}{g_{ph}^2 m} = \int_0^\infty dx \left(\frac{\tanh\sqrt{x^2 + \rho_{ch}^2/4T^2}}{\sqrt{x^2 + \rho_{ch}^2/4T^2}} - \frac{1}{x + \omega_0/2T} \right). \tag{26}$$

On the other hand, the condition $\rho_{ch} = 0$ in (26) leads to the equation

$$\frac{2\pi}{g_{ph}^2 m} = \int_0^\infty dx \left(\frac{\tanh x}{x} - \frac{1}{x + \omega_0/2T_{\rho_{ch}}} \right). \tag{27}$$

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[3]Being equal (in mean field approximation only) these temperatures (T_ρ and T_{BCS}^{MF}) are in fact different: if T_{BCS}^{MF} immediately falls down to zero as fluctuations are taken into account, T_ρ does not and is renormalized only.

for T_ρ. From (26) and (27) it directly follows that

$$\int_0^\infty dx \left(\frac{\tanh x}{x} - \frac{\tanh \sqrt{x^2 + \rho_{ch}^2/4T^2}}{\sqrt{x^2 + \rho_{ch}^2/4T^2}} \right) = \ln \frac{T_\rho}{T}.$$

Then using the approximation

$$\frac{\tanh \sqrt{x^2 + \rho_{ch}^2/4T^2}}{\sqrt{x^2 + \rho_{ch}^2/4T^2}} \simeq \begin{cases} 1 - 3^{-1} \left[x^2 + \rho_{ch}^2/4T^2 \right], & x \leq 1; \\ x^{-1} - \rho_{ch}^2/8T^2 x^3, & x > 1, \end{cases}$$

one directly comes to the expression needed:

$$\rho_{ch}(T) \simeq 2.62 T_\rho \sqrt{T_\rho/T - 1}. \tag{28}$$

Recall that the generally accepted 3D result is $\Delta_{BCS}(T) = 3.06 T_{BCS}^{MF} \sqrt{T_{BCS}^{MF}/T - 1}$ [17] and this small difference can be explained by the above approximation what, however, is suitable for the following below qualitative discussion (see next Section).

The dependence (28) has to be substituted in equation (22). And again because of $\mu/2T_{BKT} \simeq \epsilon_F/2T_{BKT} \gg 1$ and $\rho_{ch}(T_{BKT})/2T_{BKT} \ll 1$ when $T_{BKT} \to T_\rho$ this equation can be written as

$$\frac{\epsilon_F}{4T_{BKT}} \left[1 - \frac{\rho_{ch}^2}{4T_{BKT}^2} \frac{\partial}{\partial(\rho_{ch}^2/4T_{BKT}^2)} \right] \int_0^\infty dx \left(\frac{1}{\cosh^2 x} - \right.$$
$$\left. \frac{1}{\cosh^2 \sqrt{x^2 + \rho_{ch}^2/4T_{BKT}^2}} \right) = 1. \tag{29}$$

At last, using expansion in $\rho_{ch}/2T_{BKT}$ in integral (29), the latter can be transformed to

$$\frac{a\epsilon_F}{8T_{BKT}} \left(\frac{\rho_{ch}}{2T_{BKT}} \right)^4 = 1, \tag{30}$$

where the numerical constant

$$a = \int_0^\infty dx \frac{\tanh^2 x - x^{-1} \tanh x + 1}{2x^2 \cosh x} \simeq 1.98.$$

Combining now (28) and (30) one comes to the final simple relation between $T_{\rho_{ch}}$ and T_{BKT} for the large carrier density:

$$T_{BKT} \simeq T_\rho (1 - 1.17 \sqrt{T_\rho/\epsilon_F}),$$

13

i.e. T_{BKT} as a function of n_f really approaches T_ρ (or T_{BCS}^{MF}) (see fig.2).

As to crossover region defined by the equality $\mu \simeq 0$ it is easy to convince from the equations (21)-(24) and fig.2 that it corresponds to the densities when the temperatures T_ρ are essentially different. It is important that because of relatively low for the phonon case value of the bound pair-state energy and so very small region of negative n_f [16], the behaviour $T_{BKT}(n_f) \sim \epsilon_F$ hardly corresponds to Bose-Einstein condensation and in fact takes place at $\mu > 0$.

4) $g_{4F} < 0, g_{ph} \neq 0$, but $g_{ph}^2 >> |g_{4F}|$.

This condition provides the total fermion-fermion attraction channel only, so $\rho_{ch} \neq 0$ (see below). As it was said above the local 4F-repulsion qualitatively can correspond to the screened Coulomb repulsion. In this situation the cut parameter must be introduced to avoid the divergence in (23). It is interesting to consider two situations: i) the cut parameter (the boundary Coulomb frecuency ω_c) goes to infinity (i.e. small concentration or local pairing) and ii) when ω_c is large but finite $\omega_c >> \omega_0$ but $g_{ph} >> |g_{4F}|$. Probably it is the situation what is intimately related to the real HTSCs (and superconductors at all).

i) The case 1) is restored in general but the effective 4F-constant $g_{4F}^{**} < 0$. It is important that T_{BKT} preserves its linear assymptotics at small n_f.

ii) In this case g_{4F} in (23) can be substituted by $g_{4F}D(i\omega_n)$ with $\omega_0 = \omega_c$ (the propagator D is defined by (2)). The situation is similar to the case 3) but the effective coupling constant is smaller. Such a decreasing leads to the narrowing of the abnormal normal phase region because of lowering of the temperature T_ρ. In particular, it is not difficult to obtain the well known Tolmachev logarithmic correction to T_ρ (see, for example, [22]):

$$T_\rho = \frac{2\gamma\omega_0}{\pi} exp(-\frac{1}{g_{ph}^2 m/2\pi - \mu^*}),$$

where $\mu^* = g_{4F}N(0)/(1 + g_{4F}N(0)\ln(\epsilon_F/\omega_0))$ ($N(0)$ is the density of states at the Fermi surface).

4   Conclusion

The model proposed to describe the possible two-stage superconducting phase transition in 2D (and quasi-2D) metallic systems is in fact very simplified in order to investigate their most typical and general features. All the more surprisingly that it catches some essential details which are characteristic for underdoped HTSC copper oxides. In particular, the experimental data demonstrates [28, 29] that i) indeed for low n_f the critical temperature T_c is proportional to n_f (what is simply ϵ_F), ii) T_c shows saturation when n_f approaches so called "optimal doping" (i.e. carrier concentration when T_c as function of n_f reaches its highest possible in given compound value), iii) the ratio T_c/ϵ_F in these and other "exotic" superconductors is as high as $10^{-2} - 10^{-1}$ what independently points out on rather small Fermi energy, etc (for details see [29]).

One would think that the pecularities mentioned receive their natural explanation on the basis of the model of metal with indirect inter-fermion interaction if the temperature T_{BKT} is implied as critical one T_c (this is justified for pure 2D systems [30]). In quasi-2D model because of the third spatial direction and the phase fluctuation stabilization the true temperature of ordinary homogeneous ordering arises [29, 31] (see also [11]).

As regards the second temperature, T_ρ, it usually introduced by empirically as some temperature point T^* where observable spectral (or magnetic) properties of HTSC begin to deviate appreciably from their standard for normal metallic state behaviour [5, 6, 7, 8, 9]. As a rule such a deviation is connected with appearance of fluctuating (short-living) pairs. We, however, showed that some finite number of these pairs does exist and is to formed at definite temperature due to phase transition between normal and pseudogap (also normal) phases. The only difference with supposed dependence T^* on the density of doped holes consists of assymptotics at n_f decreasing: we have obtained that this temperature is also reduced while usually (see, for example, [29]) T^* is presumably depicted as such one that incriases with n_f decreases. It seems that the latter has no satisfactory grounds. Nevertheless it must be stressed that the above limit $T_\rho(n_f) \rightarrow 0$ when $n_f \rightarrow 0$ can not be also considered as sufficiently regular because of the growth here of the neutral OP fluctuations contributions

of which was not taken into account and which become rather important at small n_f.

At last the model under consideration qualitatively correctly describes the explicit narrowing of the pseudogap phase area at carrier density increase (such a diminution results in rather rapid reapproachment the temperature $T_c(T_{BKT})$ and T_ρ and their experimental confluence (indistinguishableness) in BCS limit.

Some important problems remain open and are to be solved. Among them there are: more complete and deep development of the model which has to consider different kinds of the dispersion laws for the intermediate bosons; more careful taking into account of the Coulombic repulsion; neutral OP fluctuations, especially at low n_f; generalization of the approach on the case of non-isotopic pairing. On the other hand, high-T_c compounds must be investigated in the frame of more realistic model that such their pecularities as magnetism of cuprate layers, non-qudratic free carrier dispersion law with van Hove singularities in the hole density of states. One of the most interesting problem is to obtain doping and temperature effective action which is equivalent to Ginzburg-Landau potential because in many cases the phenomenology is more preferable.

References

[1] V.M. Loktev, Sov. Low Temp. Phys. **22**, 1 (1996).

[2] D.S. Marshall, D.S. Dessau, A.G. Loeser et al., Phys. Rev. Lett. **76**, 4841 (1996).

[3] H. Ding, T. Yokoya, J.C. Campuzano, Nature **382**, 51 (1996).

[4] A.G. Loeser, Z.-X. Shen, D.S. Dessau et al., Sciences **273**, 325 (1996).

[5] B.G. Levi, Physics Today **49**, 17 (1996).

[6] B. Batlogg and V.J. Emery, Nature **382**, 20 (1996).

[7] N.P. Ong, Science **273**, 321 (1996).

[8] C. Berthier, M. Julien, M. Horvatic and Y. Berthier, J.Phys. I (France) **6**, 2205 (1996).

[9] D. Pines, Tr. J. of Physics **20**, 535 (1996).

[10] M. Randeria, in Bose-Einstein Condensation, edited by A. Griffin, D.W. Snoke and S. Stringari, P.355 (Cambrige U.P., NY, 1995).

[11] V.M. Loktev and S.G. Sharapov, Cond. mat./ 9706285 (1997).

[12] E.V. Gorbar, V.P. Gusynin and V.M. Loktev, Superconductivity: Physics, Chemistry, Technology **6**, 375 (1992); Sov. Low Temp. Phys. **19**, 832 (1993).

[13] R. MacKenzie, P.K. Panigrahi and S.Sakhi, Int. J. Mod. Phys. A **9**, 3603 (1994).

[14] M. Drechsler and W. Zwerger, Ann. Phys. **1**, 15 (1992).

[15] V.P. Gusynin, V.M. Loktev and S.G. Sharapov, Pis'ma v ZhETF **65**, 170 (1997).

[16] V.M. Loktev, V.M. Turkowski and S.G. Sharapov, Journ. of Phys. Stud. **3**, 311 (1997).

[17] A.A. Abrikosov, L.P. Gor'kov and I.E. Dzyaloshinskii, Methods of Quantum Field Theory in Statistical Physics (Prentice-Hall, Englewood, NJ, 1963).

[18] E.A. Pashytsky, Sov. Low. Temp. Phys. **21**, 995 (1996); *ibid* **21**, 1019 (1996).

[19] V.L. Ginzburg, Usp. Fiz. Nauk **167**, 229 (1997).

[20] Y. Nambu, Phys.Rev. **117**, 648 (1961).

[21] T. Kugo, Preprint-KUNS 1086 HE(TH)91/12 July (Kyoto, Japan, 1991).

[22] S.V. Vonsovskii, Yu.A. Izyumov, E.Z. Kurmaev, Superconductivity of transition metals, their alloys and compounds (Nauka, Moscow, 1977).

[23] J.R. Schrieffer, Theory of superconductivity (Benjamin, NY, 1964).

[24] B.T. Geilikman, Usp. Fiz. Nauk **88**, 327 (1966).

[25] J.M. Cornwall, R. Jackiw and E. Tomboulis, Phys. Rev. D **10**, 2428 (1974).

[26] L.J.R. Aitchison, P. Ao, D.J. Thouless, and X.-M. Zhu, Preprint CERN- TH.7385/94 (1994).

[27] Yu.A. Izyumov and Yu.N. Skryabin, Statistical mechanics of magnetically ordered systems (Nauka, Moscow, 1987).

[28] Y.J. Uemura, Nature **364**, 605 (1993).

[29] Y.J. Uemura, Cond. mat./9706151 (1997).

[30] R. MacKenzie, P.K. Panigrahi and S. Sakhi, Int.J.Mod.Phys.A **9**, 3603 (1994).

[31] E.V. Gorbar, V.M. Loktev, S.G. Sharapov, Physica C **257**, 355 (1996).

Fig.1

Phase $T - n_f$ diagram of 2D metal with 4F fermion attraction. The lines correspond to the functions $T_\rho(n_f)$ (the upper line) and $T_{BKT(n_f)}$ (the lower one) at $\lambda = 0.5$. The figures I, II and III show the regions of the normal, abnormal normal (pseudogap) and superconducting phases, respectively.

Fig.2

Phase $T - n_f$ diagram of 2D metal with indirect inter-carrier attraction. Similarly to the Fig.1, the lines correspond to the functions $T_\rho(n_f)$ and $T_{BKT(n_f)}$ and separate the same regions ($\lambda = 0.5$).